

MERIT

A Monotonically Error-Bound Improving Technique for Mathematical Optimization

- It's about a computational approach to derive sub-optimality guarantees.
- You want to know how much the solution can be trusted . . .

The Central Idea

Let $\mathcal{P}(v, x)$ be an optimization problem *structure* with given and optimization variables partitioned as (v, x) .

Example:

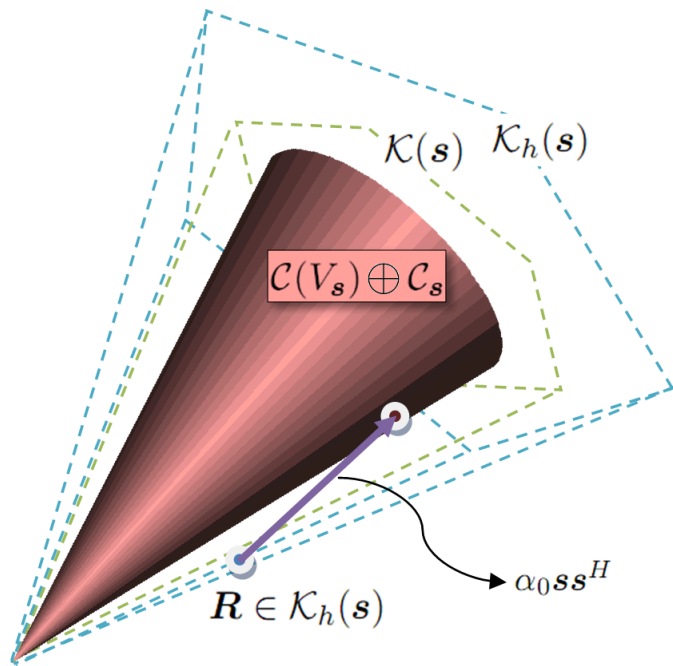
$$\begin{array}{ccc} \mathbf{X} = \arg \max & \text{tr}(\mathbf{R}\mathbf{X}) & \\ \text{s.t.} & \text{tr}(\mathbf{Q}\mathbf{X}) \leq t & \end{array} \quad \begin{array}{c} \text{variable partitioning} \\ \implies \end{array} \quad \begin{array}{l} \mathbf{R}, \mathbf{Q}, t \rightarrow v \\ \mathbf{X} \rightarrow x \end{array}$$

Now suppose $\mathcal{P}(v, x)$ is a “difficult” optimization problem; however,

- A sequence v_1, v_2, v_3, \dots of v can be constructed such that the associated global optima of the problem, viz. $x_k = \arg \max_x \mathcal{P}(v_k, x)$ are known for any v_k , and the “distance” between v and v_k , is decreasing with k .
- A sub-optimality guarantee of the obtained solutions x_k can be efficiently computed using the distance between v and v_k .

Then, **computational sub-optimality guarantees** can be obtained along with the approximate solutions, that might

- outperform existing analytically derived sub-optimality guarantees, or
- be the only class of sub-optimality guarantees in cases where no a priori known analytical guarantees are available for the given problem.



An illustration of the cone approximation technique used for MERIT's derivation in unimodular quadratic programming. See [Applications](#) for details.